Project Report on Adaptive Resource Allocation Algorithm for Multiuser Mimo-Ofdm Systems:

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Abstract

The multiuser MIMO-OFDM system has great potential of providing enormous capacity due to its integrated space-frequency diversity and multiuser diversity. Assuming the knowledge of channel state information (CSI) to be available at the transmitter, the performance can be further improved through the adaptive resource allocation. For the OFDMA systems with single antenna, several resource allocation methods were proposed in literature to minimize the total transmit power given QoS by utilizing multiuser diversity in frequency domain. In this article we review from literature one such optimal resource allocation algorithm for multi user SISO-OFDM systems (Downlink scenario) which guarantees a solution that satisfies the proportional rate constraint in the strictest sense and then extend this algorithm to multi user MIMO systems and review results obtained.
**Introduction:**

With a drastic increase in number of users feeling comfortable while accessing wireless services in last few years, there is a fair demand for higher system throughputs to accommodate more users with higher data rates. It is illustrious that Multiple-Input Multiple-Output (MIMO) based Orthogonal Frequency Division Multiplexing (OFDM) has the potential to achieve high system capacity and transmit-receive diversity for reliable communication links of any wireless system, hence is considered as the future of wireless communication systems. MIMO OFDM systems provide higher data rates, support a large number of users with flexibility in Quality of Service (QoS) and provide high quality transmission in comparison with the existing ones. But in order to fulfil these requirements some constraints have to be very well addressed such as limited availability of frequency spectrum, availability of total transmit power and nature of wireless channels[5].

Power and subcarrier allocation schemes for SISO-OFDM (OFDMA) systems in multiuser downlink scenario are very well acknowledged and documented in past. However the resource scheduling strategy for downlink multi user MIMO-OFDM scenario is still found seldom in literature. Most of the recent works in literature [4]-[6] have been extending these concepts of SISO-OFDM to MIMO-OFDM systems. Let’s first look at FDM:

![FDM Diagram]

In FDM system, signals from multiple transmitters are transmitted simultaneously (at the same time slot) over multiple frequencies. Each frequency range (sub-carrier) is modulated separately by different data stream and a spacing (guard band) is placed between subcarriers to avoid signal overlap.

![OFDM Diagram]

OFDM is sometimes referred to as discrete multi-tone modulation because, instead of a single carrier being modulated, a large number of evenly spaced subcarriers are modulated using some m-ary of QAM. This is a spread-spectrum technique that increases the efficiency of data communications by increasing data throughput because there are more carriers to modulate. In addition, problems with multi-path signal cancellation and spectral
interference are greatly reduced by selectively modulating the “clear” carriers or ignoring carriers with high bit-rate errors.

Like FDM, OFDM also uses multiple sub-carriers but the sub-carriers are closely spaced to each other without causing interference, removing guard bands between adjacent sub-carriers. This is possible because the frequencies (sub-carriers) are orthogonal; meaning the peak of one sub-carrier coincides with the null of an adjacent sub-carrier. In an OFDM system, a very high rate data stream is divided into multiple parallel low rate data streams. Each smaller data stream is then mapped to individual data sub-carrier and modulated using some sorts of PSK (Phase Shift Keying) or QAM (Quadrature Amplitude Modulation). i.e. BPSK, QPSK, 16-QAM, 64-QAM. OFDM needs less bandwidth than FDM to carry the same amount of information which translates to higher spectral efficiency. Besides a high spectral efficiency, an OFDM system such as WiMAX is more resilient in NLOS environment. It can efficiently overcome interference and frequency-selective fading caused by multipath because equalizing is done on a subset of sub-carriers instead of a single broader carrier. The effect of ISI (Inter Symbol Interference) is suppressed by virtue of a longer symbol period of the parallel OFDM sub-carriers than a single carrier system and the use of a cyclic prefix (CP).

The OFDM spread-spectrum scheme is used for many broadly used applications, including digital TV broadcasting in Australia, Japan and Europe; digital audio broadcasting in Europe; Asynchronous Digital Subscriber Line (ADSL) modems and wireless networking worldwide (IEEE 802.11a/g).

Like OFDM, OFDMA employs multiple closely spaced sub-carriers, but the sub-carriers are divided into groups of sub-carriers. Each group is named a sub-channel. The sub-carriers that form a sub-channel need not be adjacent. In the downlink, a sub-channel may be intended for different receivers. In the uplink, a transmitter may be assigned one or more sub-channels. Sub channelization defines sub-channels that can be allocated to subscriber stations (SSs) depending on their channel conditions and data requirements. Using sub channelization, within the same time slot a Mobile WiMAX Base Station (BS) can allocate more transmit power to user devices (SSs) with lower SNR (Signal-to-Noise Ratio), and less power to user devices with higher SNR. Sub channelization also enables the BS to allocate higher power to sub-channels assigned to indoor SSs resulting in better in-building coverage. Sub channelization in the uplink can save a user device transmit power.
because it can concentrate power only on certain sub-channel(s) allocated to it. This power-saving feature is particularly useful for battery-powered user devices, the likely case in Mobile WiMAX.

![Frequency](image)

OFDM (orthogonal frequency division multiplexing) is based on the concept of multicarrier transmission. This scheme divides the broadband channel into $N$ narrowband sub channels, each with a bandwidth much smaller than the coherence bandwidth of the channel. The high rate data stream is then split into $N$ substreams of lower rate data which are modulated into $N$ OFDM symbols and transmitted simultaneously on $N$ orthogonal subcarriers. The low bandwidth of the sub channels along with the frequency spacing between them are necessary to have flat fading orthogonal subcarriers with approximately constant channel gain during each transmission block [5]. OFDMA is a multi-user OFDM that allows multiple access on the same channel (a channel being a group of evenly spaced subcarriers).

MIMO (multiple input, multiple output) is an antenna technology for wireless communications in which multiple antennas are used at both the source (transmitter) and the destination (receiver). The antennas at each end of the communications circuit are combined to minimize errors and optimize data speed. A network design incorporating MIMO technology provides the scalability needed to quickly deliver multimedia content to the mass market. MIMO works by creating multiple parallel data streams between the multiple transmit and receive antennas. Using the multi-path phenomenon, it can differentiate the separate signal paths from each MIMO antenna [4]. Like any other communication system MIMO-OFDM system also has transmitter and receiver but the antennas are more than one, both at transmitting and receiving end. MIMO system can be implemented in various ways, if we need to take the diversity advantage to combat fading then we need to send the same signals through various MIMO antennas and at the receiving end all the signals received by MIMO antennas will receive the same signals travel through various path. In this case the entire received signal must pass through un-correlated channels. If we are concerned to use MIMO for capacity increase then we can send different set of data using a number of antennas and the same number of antennas will receive the signals in the receiving end. For MIMO to be efficient antenna spacing need to be done very carefully- at least it must be half the wave length of the transmitting signal [5].

In a fixed TDMA scheme where time slots are non adaptively assigned, users who have good channel responses can reliably receive higher data rates while others suffer from poor channel responses. With large path loss, this discrepancy cannot be ignored, and the system becomes unfair for the users with poor channel gains [2].
The problem of sub-channel and power allocation for a multiuser Orthogonal Frequency Division Multiplexing (OFDM) system, while maximizing the total system throughput and satisfying the typical constraints of total power and fairness can be modelled as a mixed binary integer programming problem[1]. The optimal solution for this problem is generally hard to find. The typical approach is to utilize a sub-optimal sub-channel allocation algorithm and then obtain the optimal power distribution for that specific sub-channel allocation [2].

We would initially consider reviewing the literature for an algorithm devised for a multiuser OFDM scheme. In [8], the authors considered extending OFDM adaptive modulation technique to multiuser frequency selective fading situations. In this multiuser subcarrier, bit, and power allocation scheme all users transmit in all time slots. When a set of user data rates are given, the algorithm attempts to minimize transmit power under a fixed performance requirement. It optimizes the algorithm by minimizing the overall transmit power by allocating the subcarriers to the users and by determining the number of bits and the power level transmitted on each subcarrier based on the instantaneous fading characteristics of all users.

A major emphasis in recent research has been given to a deeper analysis of extending the multi user OFDM systems to MIMO-OFDM systems. Now, we would focus specifically on the research done in literature for multiuser MIMO-OFDM systems using adaptive subcarrier or power allocation algorithms. Most of the algorithms [7] & [9], in literature propose scheduling schemes that prefer dynamic subcarrier allocation and some algorithms [10], use Lagrange multiplier technique (one step optimal solution). By using various techniques all the algorithms situate their respective efforts in minimizing the overall transmit power for a given BER, data rate and QoS requirement target value, or increase the system capacity with low computation complexity.

There are two classes of optimization techniques that have been proposed in literature for adaptive multiuser systems, (a) minimum transmit power and (b) maximum system capacity with constrained overall transmit power. But for optimal scheduling performance in multiuser MIMO-OFDM systems, the complexity of system increases exponentially with number of subcarrier, multiuser and transmit antenna. Consequently a low complexity suboptimal scheduling algorithm has been a major research objective in recent years [7].

Compared to single-user MIMO in distinction from the multiuser type, multiuser MIMO (MU-MIMO) achieves higher transmission capacity in the system as a whole with simple terminals. However, the technological hurdles become progressively higher. In multiuser MIMO networks, the spatial degrees of freedom offered by multiple antennas can be advantageously exploited to enhance the system capacity, by scheduling multiple users to simultaneously share the spatial channel [4]. From the concepts of information theoretic studies it can be said that resource allocation techniques help us in exploiting the gains of multiuser MIMO systems.
It is well known that deploying multiple antennas at the transmitter and/or receiver will improve the performance and capacity considerably. Some of the works in literature focus on the development of criteria which was used to allocate a subcarrier among users and present an adaptive multiuser MIMO-OFDM scheme combined beamforming, adaptive subcarrier and bit allocation [8].

But we would center majorly on the algorithm proposed in [1], which proposes an efficient resource allocation algorithm that assigns optimal power for a given sub-channel allocation scheme. The algorithm proposed does not make any assumptions about the channels or proportionality constants. Moreover, the proposed algorithm satisfies the proportional rate constraint in the strictest sense. It proposes an algorithm that is based on the optimal solution proposed in [3] where the proportional rates constraint (see section II) is satisfied in the strictest sense (i.e. in every time slot) and hence obtains absolute guarantees for the expected quality of service (QoS). In addition, the proposed solution is valid for any arbitrary channel conditions and does not make any assumptions for the sub-channel gains and the proportionality ratios used in the fairness constraint.

The rest of this article is organized as follows: Section II defines the problem and the analytical model for OFDMA systems. Various suboptimal & optimal algorithms proposed for the problem defined are discussed and the results of [1] are reviewed in section III, while section IV extends this scheduling algorithm to MIMO systems, section V provides and discusses results for both cases. Finally, the paper concludes in section VI.
PROBLEM DEFINATION

For a total system bandwidth which is to be divided into \( N \) narrowband flat fading subchannels to serve \( K \) users, it is desirable to distribute the \( N \) sub-channels amongst the \( K \) users using the sub-channel allocation algorithm. Let \( H_{k,n} \) be the ratio of \( n^{th} \) Sub channel Power Gain to Noise power as received by \( k^{th} \) user where \( n=1,2,3,...,N \) & \( k=1,2,3,...,K \).

![Figure: An OFDMA subchannel frequency division is shown in figure.](image)

The power allocations should be done in a way, that they maximize the overall network throughput accounting for the total power constraint (less than the total power budget for the system), subchannel allocations constraint for different users (considered to be mutually exclusive) and also the fairness constraint( i.e. proportionality rate constraints predefined by the service providers). For Subchannel (frequency) allocation of the \( k^{th} \) user denoted by \( \Omega_k \), the power Allocation in order to maximize overall network throughput is specified by maximizing the Shannon capacity for SISO channels:

\[
\max_{p_{k,n}} \sum_{k=1}^{K} \sum_{n \in \Omega_k} \frac{1}{N} \log_2(1 + P_{k,n}H_{k,n})
\]

While being subjected to following constraints:

\[ \sum_{k=1}^{K} \sum_{n \in \Omega_k} p_{k,n} \leq P_{total} \text{ and } p_{k,n} \geq 0. \]
Where $P_{\text{total}}$ is the total power budget for the system and $p_{k,n}$ is power allocation for $H_{k,n}$ subchannel.

The subchannel allocations $\Omega_k$'s for different users are mutually exclusive, disjoint, and

$$\Omega_1 \cup \Omega_2 \cup \Omega_3 \ldots \cup \Omega_k \subseteq \{1, 2, \ldots, N\}$$

The proportional rate constraints are to be satisfied for a promised QoS,

$$\frac{R_1}{\gamma_1} = \frac{R_2}{\gamma_2} = \ldots = \frac{R_K}{\gamma_K}.$$

Where $R_k$ is $K$th user bit rate given by $\sum_{n \in \Omega_k} \frac{1}{N} \log_2 \left(1 + \frac{P_{k,n} H_{k,n}}{N_k}\right)$, when the allocation process is completed, and $\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_k$ are the proportional rates constants based on the specified QoS parameters promised for the users. (Specifying the relative bit rate enjoyed by users.)

The usual method for solving such optimization problem along with the corresponding constraints is to use the Lagrange multipliers as in [3], and construct a cost function using these Lagrange multipliers which is differentiated & made equal to zero [1], yielding:

$$\frac{1}{\gamma_1} \frac{N_1}{N} \left( \log_2 \left(1 + H_{k,1} \frac{P_{k,1}}{N_1} - V_k \right) + \log_2 W_k \right) = \frac{1}{\gamma_k} \frac{N_k}{N} \left( \log_2 \left(1 + H_{k,k} \frac{P_{k,k}}{N_k} - V_k \right) + \log_2 W_k \right)$$

$$\rightarrow (1)$$

The Total power assigned to the $k$th user is given by $P_{k,\text{Total}} = \sum_{n=1}^{N_k} p_{k,n}$, the constants $V_k$ and $W_k$ are given by:

$$V_k = \sum_{n=2}^{N_k} \frac{H_{k,n} - H_{k,1}}{H_{k,n} H_{k,1}},$$

$$W_k = \left( \prod_{n=2}^{N_k} \frac{H_{k,n}}{H_{k,1}} \right)^{\frac{1}{N_k}}$$

These constants depend only on allocate subcarrier terms $\Omega_k$'s and are only for the materializing subchannel frequency algorithm. The cost function assumes that the channel power gains for each user satisfy $H_{k,1} \leq H_{k,2} \leq \ldots \leq H_{k,N}$. This implies that the number of
elements in set $\Omega_k$ is equal to number of subchannels allocated for $k$th user i.e. $N_k$, and quantity $V_k$ being positive always.

Use of equally weighted capacity sum as the optimizing function, and introducing the scheme of proportional fairness into the system (by adding a set of nonlinear constraints) gives a benefit of explicitly controlling the capacity ratios among various users, while ensuring each user his target data rate.

Using the derived cost function the total power allocation ($P_{k,total}$) for a particular user can be found, which gives the power allocations for the individual subchannels, calculated as:

$$p_{k,1} = (P_{k,Total} - V_k) / N_k$$

$$p_{k,n} = p_{k,1} + \frac{H_{k,n} - H_{K,1}}{H_{k,n} H_{K,1}}$$

The cost function derived for the has a set of $(K-1)$ simultaneous non-linear equations, which are used to calculate $P_{k,total}$ and $p_{k,n}$ in order to achieve maximum throughput and satisfy various constraints (QoS, data rate etc).
Various methods proposed to obtain an sub-optimal or optimal solution

Although there are many articles in literature which discuss this optimization issue and try to provide an algorithm that attempts to solve the problem in a sub-optimal manner or optimal in some cases, but very few papers have proposed an algorithm that provides an optimal solution for the problem while satisfying the proportional rate constraints in strict sense.

These algorithms can be classified majorly as rate adaptive and margin adaptive. Rate adaptive algorithms use a utility function (cost function) in order to accomplish both efficiency and fairness, where one has an option of selecting a marginally decreasing cost function that guarantees optimality of the solution. While in margin adaptive algorithms a given set of user data rates are assumed for a fixed QoS requirement. However there are few shortcomings of these margin adaptive procedures, as the subcarriers are grouped neglecting the frequency diversity for a flat channel [5]. Therefore, in this paper we discuss on some of the rate adaptive algorithms.

In 2000, Wonjong Rhee et al. [2] studied the max-min problem (i.e. maximizing the total throughput with fairness) of dynamic multiuser subchannel allocation in downlink of OFDM systems, with some assumptions about the channel (quasi-static) and came up with an highly efficient and very simple, low complexity adaptive sub channel allocation algorithm that maximizes the worst user’s capacity, assuring all users the same data rate defying the effect of large path loss and random fading, which would been resulting in poor channel gains for some set of users.

For a total bandwidth of B, N number of subchannels, additive white Gaussian noise of power $N_0$. The algorithm proposed in [2], aims at maximizing the total rate within the total power constraint adopting a new parameter $\omega_{k,n}$ representing portion of subchannel n assigned to user k,

$$\max_{P_{k,n}, \omega_{k,n}} \min_k \sum_{n=1}^{N} \frac{B \cdot \omega_{k,n}}{N} \log_2 \left( 1 + \frac{P_{k,n}H_{k,n}^2}{N_0 \omega_{k,n} N} \right)$$

While maintaining rate proportionality among the users indicated by $R_1$: $R_2$: .......: $R_K = \gamma_1$: $\gamma_2$: .......: $\gamma_K$. Where $\gamma_1, \gamma_2, \gamma_3, ......., \gamma_K$ are a set of predetermined proportional rate constraints (positive & real no’s) as discussed in section II. The minimum value, $\gamma_{\min} = 1$, is for the user with least required proportional rate. In the algorithm proposed all the proportional rate constraints are considered to be equal. Preferably the power and subcarrier allocation
should be carried out jointly (for optimal solution) which would give rise to high computational complexity, forcing one to go for an suboptimal solution to solve the utility function defined in section II.

The suboptimal algorithm proposed in [2] demonstrates 50 to 130% of capacity gain over a non-adaptive TDMA resource allocation scheme and less than 4% spectral efficiency loss in contrast to optimal solution with adaptive power allocation. It achieves acceptable fairness only if the number of subcarriers is much larger than the number of users [2] i.e., \( N \gg K \).

Although acceptable fairness amongst users is achieved in [2], the frequency selective nature of a user's channel is not fully utilized by allocating power uniformly across all subcarriers. To improve its performance, in 2005, Shen et al. [3] added a second step of adaptive power allocation to further insist on the rate proportionality among different users. The two-step approach adopted in [3] is as follows: in the first step, the modified version of the algorithm outlined in [2] is employed for subcarrier allocation to achieve proportional fairness. Hence, instead of giving priority to the user with the least achieved data rate \((R_k)\), priority is given to the user with the least achieved proportional data rate i.e. \( R_K \gamma_K \).

For a total bandwidth of \( B \), \( N \) number of subchannels, additive white Gaussian noise of power \( N_0 \), the proposed algorithm aims at maximizing:

\[
\max_{p_{k,n}} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} \log_2 \left( 1 + \frac{p_{k,n} h_{k,n}^2}{N_0 B N} \right),
\]

(Where \( p_{k,n} \) can take only the value 0 or 1, indicating whether the subchannel \( n \) is used by \( k \) or not) being subject to various constraints listed in section II including the proportional rate constraint \( (R_1: R_2: \ldots \ldots : R_K = \gamma_1: \gamma_2: \ldots \ldots : \gamma_K) \). Here, the capacity for user \( k \) is defined as:

\[
R_k = \sum_{n=1}^{N} \frac{p_{k,n}}{N} \log_2 \left( 1 + \frac{p_{k,n} h_{k,n}^2}{N_0 B N} \right).
\]

In first step of the algorithm proposed [3], the achieved rate is calculated considering equal power on all the subcarriers, after subcarrier allocation is carried out, the problem is reduced to maximization over continuous variables of power. In the second step, the power is reallocated between the users and then among the subcarriers through the use of water-filling to impose the rate proportionality among all users.

The suboptimal/ optimal algorithms described above, either use fixed power allocation and perform only subcarrier allocation [2], or handle subcarrier and power allocation separately [3] to reduce the complexity of the algorithm, not necessarily satisfying the
proportional rates constraint for the general case. However, subcarrier and power allocation have to be carried out jointly to achieve the optimal solution.

In 2008, Ashraf et al.[1] proposed an efficient algorithm that solves the utility function without making any assumption about the channel power gains, and ensures that the final \( p_{kn} \)’s satisfy the proportional rates constraint in the strict sense. This algorithm [1] solves the (K-1) non linear equations obtained from the cost function obtained in section II by defining a new parameter \( X_k \), given by

\[
X_k = 1 + H_{k,1} \left( P_{K,total} - V_k \right) / N_k.
\]

Thus total power for each user is \( P_{k,total} = V_k + N_k \left( X_k - 1 \right) / H_{k,1} \). By substituting this parameter \( X_k \) in cost function, we obtain:

\[
X_k = \frac{\left( X_i W_i \right)^{r_k N_i / y_i N_k}}{W_k}, \forall i, k \in \{1, 2, ..., K\} \rightarrow (2)
\]

To obtain \( X_i \) the above equation is is and the total power constraint is used, deriving

\[
\sum_{k=1}^{K} \left( V_k + \frac{N_k}{H_{k,1}} \left( (X_i W_i)^{r_k N_i / y_i N_k} / (W_k - 1) \right) - P_{total} = 0 \rightarrow (3)
\]

Therefore, the algorithm proposed is discussed here in brief, as our aim to extend this algorithm to multiple antenna multiuser system:

1) For a given set of sub-channel frequency allocations \( \Omega_k \forall k = 1, 2, ..., K \), corresponding \( V_k \) and \( W_k \) are calculated.

2) Then the inequality: \( \sum_{k=1}^{K} V_k \leq P_{total} \) is checked, If the inequality is not satisfied then a set of \( \Omega_k \) are selected that correspond to the largest \( V_k \) (where \( k = 1, 2, ..., K \)) used to drop the channel with the smallest power gain \( H_{k,n} \), then set \( \Omega_k \) are updated, and corresponding \( V_k \) and \( W_k \) are recalculated, thus the above inequality is checked again.

3) If the inequality is not satisfied, User index \( i \) is selected such that corresponding \( (W_i)_{\frac{N_i}{y_i}} \geq (W_i)_{\frac{N_k}{y_k}} \) for all \( k \neq i \) and \( k = 1, 2, ..., K \). The theoretical possible range for \( X_i \) are all values between 1 and \( 1 + H_{i,1} \left( P_{K,total} - V_i \right) / N_i \). Then if eqn (1) has different signs when \( X_i \) assumes the two extreme values of its range then there is a valid solution \( X_i \) between the corresponding extreme values, otherwise the sub-channel frequency allocation sets are updated again as in previous step.

4) When a valid solution for equation (2) is guaranteed, It is solved for calculating \( X_i \), which is instead used for finding all \( X_k \forall k \neq i \) and \( k = 1, 2, ..., K \).
5) Therefore the corresponding total user power allocation $P_{k_{\text{total}}} \forall k = 1, 2, ..., K$ is evaluated using the definition of $X_k$. Also the individual sub-channel power allocations $p_{k,n} \forall n \in \Omega_k$ are computed.

Thus the obtained solution maximizes the system throughput and also guarantees that the provided users rates $R_k$'s satisfy the proportional rates constraint such that $R_1: R_2: ...: R_K = \gamma_1: \gamma_2: ...: \gamma_K$.

Analyzing results in [1]: The results in [1] are obtained using a 6 tap channel model, with $P_{\text{total}}$ of 1 Watt, for a Noise PSD of -65 dBW per Hz. The minimum user capacity (in bps/Hz) is for a 64 sub channels in 1MHz bandwidth configuration.

Fig.1 Minimum user capacity for multiuser OFDM vs. number of users for algorithms proposed in [1] and [3] respectively.
For the purpose of performance evaluation, Jain's Fairness index is also calculated:

\[ F = \left( \frac{\sum_{k=1}^{K} \Gamma_k}{K \sum_{k=1}^{K} \Gamma_k^2} \right)^2 \; \text{Where} \; \Gamma_k = \frac{R_k}{\gamma_k}. \]

If the proportionality rate constraints are satisfied in strict sense then all \( \Gamma_k \)'s equal to 1. Therefore fairness index nearly equals 1[5]. This fairness index is plotted for comparison in figure 2 below.

![Fairness index and adherence to proportional rates constraint for algorithms proposed in [1] and [3] respectively.](image)

Fig.2. Fairness index and adherence to proportional rates constraint for algorithms proposed in [1] and [3] respectively.

From the above results it can be derived that the algorithm proposed in [1] satisfies the proportional rates constraint in the strict sense while the actual algorithm in [3] doesn't. The results in fig. 1 also depict that the algorithm proposed in [1] has significantly higher capacity than the algorithm in [3]. Thus the algorithm devised in [1] guarantees a valid solution for optimizing the system throughput and meeting required constraints, while satisfying proportional rates constraints in strict sense.
Multiuser MIMO-OFDM system

In the downlink MIMO-OFDM system, a base station is communicating simultaneously to multiple users while both the base station and the users are equipped with multiple antennas. The problem of resource allocation in a multiuser MIMO-OFDM system is similarly formulated but is more challenging due to multiple antennas [5].

In this paper an attempt has been made to extend the algorithm proposed in [1], to a multiuser MIMO-OFDM system. Such a system uses advantage of both OFDM and MIMO, and provides diversities in various domains. In this paper we consider a multiuser MIMO-OFDM downlink scenario. It is a well known fact that we can increase capacity of a system without increasing the bandwidth and transmit power rather by just putting more antennas at transmitter and receiver side [6].

![Downlink scenario for a multiuser MIMO-OFDM system.](image)

**Channel model:** The most important part to understand and deal the MIMO capacity is the channel matrix. We generate a MIMO-OFDM channel assuming K users, $M_r$ receiving antennas and $M_t$ transmitting antennas. The frequency band is supposed to be divided into N subcarriers as in [4]. We denote the channel matrix of user $k$ on subcarrier $n$ by

$$H_{k,n} = \begin{bmatrix}
    h_{1,1}^{k,n} & \cdots & h_{1,M_t}^{k,n} \\
    \vdots & \ddots & \vdots \\
    h_{M_r,1}^{k,n} & \cdots & h_{M_r,M_t}^{k,n}
\end{bmatrix}$$

Where $h_{M_r,M_t}^{k,n}$ is channel gain from $M_t^{th}$ transmitting antenna to $M_r^{th}$ receive antenna for $k^{th}$ user over $n^{th}$ subcarrier.
Fig. 4. A typical scenario for multi user MIMO-OFDM channel as defined by the matrix above for $M_r = 4, 1&2, M_t = 4$ and $n$ users.

The sub carrier allocations should be made in a manner that they maximize (the Shannon capacity for MIMO systems):

$$\max_{p_{k,n}} \sum_{k=1}^{K} \sum_{n \in \Omega_k} \frac{1}{N} \log_2 \left( \det \left( I_{M_R} + \frac{p_{k,n}}{M_t N_0} . H_{k,n} H_{k,n}^H \right) \right);$$

Where $N_0$ is the noise power (AWGN), the subchannel allocation for the $k$th user is denoted by $\Omega_k$ and $H_{k,n}$ are the channel matrices for the respective MIMO channel existing between the transmitter and $k$th receiver. The maximization must meet the following constraints at the same time:

$\Rightarrow$ The total power constraint should be assured of,

$$\sum_{k=1}^{K} \sum_{n \in \Omega_k} p_{k,n} \leq P_{total} \text{ and } p_{k,n} \geq 0.$$  

Where $P_{total}$ is the total power budget for the system and $p_{k,n}$ is power allocation for $H_{k,n}$ subchannel.

$\Rightarrow$ Sub channel allocations $\Omega_k$'s for different users are mutually exclusive, disjoint, and

$$\Omega_1 \cup \Omega_2 \cup \Omega_3 \ldots \cup \Omega_k \subseteq \{ 1, 2, \ldots, N \}$$

$\Rightarrow$ The proportional rate constraints are to be satisfied for a promised QoS,

$$R_1/\gamma_1 = R_2/\gamma_2 = \ldots = R_K/\gamma_K.$$  

Where $R_k$ is $K$th user bit rate given by:
\[ R_k = \frac{1}{N} \log_2 \{ \det \left( I_{M_r} + \frac{P_{k,n}}{M_t N_0} \cdot H_{k,n} \cdot H_{k,n}^H \right) \}, \]

And \( \gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_k \) are the proportional rates constants based on the specified QOS parameters promised for the users.

We would assume each sub-channel as an individual channel and proceed with the utility function, as well as other assumptions made in section II for optimizing a multiple user MIMO-OFDM system. Therefore, we use the same algorithm, as in [1], and proceed by replacing its channel with a MIMO channel.
Results

The results are obtained by changing the OFDMA channel in algorithm [1] and are simulated using MATLAB by increasing number of users from 2 to 50. The frequency selective multipath channel here is modeled as one consisting of six independent Rayleigh multipaths with an exponentially decaying profile. The total power available at the base station is 1 Watt. The overall band width is 1MHz and is divided into 64 subcarriers. The power spectral density for noise is taken as 65dBW. We consider 2 x 2 and 4 x 4 MIMO-OFDM channels for all users while performing simulations i.e. Mr=2 & 4, Mt=2 & 4 respectively.

![Total Capacity Vs Users for various schemes.](image)

Figure 5: Total Capacity Vs Users for various schemes.
Fig. 6 Minimum user capacity vs. number of users for various schemes.

Figure 7: Average User capacity vs No. of users for various schemes.
The results in figures 5, 6, 7 shows that, capacity increases drastically when the channel is replaced with that of a multiuser MIMO OFDM channel. The capacity can be improved further by increasing number of receiving and transmitting antennas (as demonstrated by a 4 x 4 MIMO-OFDMA system). But as the number of users increase the capacity is reducing exponentially.
Conclusion

In this paper we have discussed some of the scheduling algorithms for solving sub carrier and power allocation problems in OFDMA and MU-MIMO OFDM systems. The numerical analysis of the optimal algorithm proposed in [1] shows that it outer performs the other sub optimal & optimal algorithms while achieving fairness constraint in strict sense.

Then the same algorithm is extended to scenario comprising a multiuser MIMO-OFDM system, although the system provides much higher capacity than that of OFDMA system capacity reduces exponentially as the number of users increase. This could be because of the limitations of the algorithm in considering some constraints pertaining to MIMO systems; also the algorithm doesn’t address the bit allocation problem in the multiuser joint optimization problem, thus failing to exploit the advantages of a MIMO-OFDM system to its maximum.

Therefore, hopefully further improvements can be observed in results by modifying the proposed algorithm to perform power, subcarrier and bit allocation simultaneously to obtain an optimal solution for the optimization problem while satisfying the proportional rate constraints in strict sense, for a multiuser MIMO-OFDM system.
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